NAG Toolbox for MATLAB

f07hu

1 Purpose

f07hu estimates the condition number of a complex Hermitian positive-definite band matrix A, where A has been factorized by f07hr.

2 Syntax

```
[rcond, info] = f07hu(uplo, kd, ab, anorm, 'n', n)
```

3 Description

f07hu estimates the condition number (in the 1-norm) of a complex Hermitian positive-definite band matrix A:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'

$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **kd – int32 scalar**

k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

3: **ab**(**ldab**,$*$) **– complex array**

The first dimension of the array **ab** must be at least **kd** $+ 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The Cholesky factor of A, as returned by f07hr.

4: **anorm – double scalar**

The 1-norm of the **original** matrix A, which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07hr or else from a copy of the original matrix A.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, work, rwork

5.4 Output Parameters

1: **rcond – double scalar**

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kd**, 4: **ab**, 5: **ldab**, 6: **anorm**, 7: **rcond**, 8: **work**, 9: **rwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07hu involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $16nk$ real floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to f07hs with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is f07hg.

9 Example

```
uplo = 'L';
kd = int32(1);
ab   =   [complex(9.39,   +0),   complex(1.69,   +0),   complex(2.65,   +0),
complex(2.17, +0);
      complex(1.08, +1.73), complex(-0.04, -0.29), complex(-0.33, -2.24),
complex(0, 0)];
anorm = 11.42943619659944;
[ab, info] = f07hr(uplo, kd, ab);
[rcond, info] = f07hu(uplo, kd, ab, anorm)
```

```
rcond =
    0.0076
info =
           0
```